**Practice Exercise lecture no. 4**

Q1.    Find the slopes of the sides of the triangle with vertices (-1, 3), (5, 4) and (2, 8).

Solution:  Let A(-1,3), B(5,4) and C(2,8) be the given points, then

Slope of side AB = $\dfrac{4-3}{5+1} = \dfrac{1}{6}$

Slope of side BC = $\dfrac{8-4}{2-5} = \dfrac{-4}{3}$

Slope of side CA = $\dfrac{3-8}{-1-2} = \dfrac{5}{3}$

Q2.    Find equation of the line passing through the point (1,2) and having slope 3.

Solution:

Point-slope form of the line passing through $P\left(x_1, y_1\right)$ and having slope $m$ is given by the equation:

$y - y_1 = m\left(x - x_1\right)$

$\Rightarrow \quad y\text{-}2 = 3\left(x\text{-}1\right)$

$\Rightarrow \quad y - 2 = 3x - 3$

$\Rightarrow \quad y = 3x - 1$

Q3.    Find the slope-intercept form of the equation of the line that passes through the point (5,-3) and perpendicular to line $y = 2x + 1$ .

Solution:

 The slope-intercept form of the line with  y-intercept b and slope m is given by the equation:  $y = mx + b$

The given line has slope 2, so the line to be determined will have slope $m = -\dfrac{1}{2}$             .

Substituting this slope and the given point in the point-slope form: $y - y_1 = m\left(x - x_1\right)$ , yields

$$y - (-3) = -\frac{1}{2}\left(x - 5\right)$$

$$\Rightarrow y + 3 = -\frac{1}{2}\left(x - 5\right)$$

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$$\Rightarrow y = -\frac{1}{2}x + \frac{5}{2} - 3 \Rightarrow y = -\frac{1}{2}x - \frac{1}{2}$$

Q4.     Find the slope and angle of inclination of the line joining the points (2, 3) and (-1, 2).

Solution:   If m is the slope of line joining the points (2, 3) and (-1, 2) then

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-3}{-1-2} = \frac{1}{3} \text{ is the slope}$$

Now angle of inclination is:

$$\tan\theta = m$$

$$\tan\theta = \frac{1}{3}$$

$$\theta = \tan^{-1}(\frac{1}{3}) = 18.43^o$$

Q5.     By means of slopes, Show that the points lie on the same line

A (-3, 4);   B (3, 2);   C (6, 1)

Solution:   Slope of line through   A(-3, 4) ; B(3, 2)   $= \frac{2-4}{3+3} = -\frac{2}{6} = -\frac{1}{3}$

Slope of line through   B(3, 2) ; C(6, 1)   $= \frac{1-2}{6-3} = -\frac{1}{3}$

Slope of line through   C(6, 1) ; A(-3, 4)   $= \frac{4-1}{-3-6} = -\frac{3}{9} = -\frac{1}{3}$

Since all slopes are same, so the given points lie on the same line.

# Practice Exercise
# Lecture No.5: Distance, Circles, Equations

**Solution 1:**

The formula to find the distance between any two points $(x_1, y_1)$ and $(x_2, y_2)$ in the coordinate plane is given as

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The given points are $(5, 6)$ and $(2, 4)$, so the distance between these two points will be
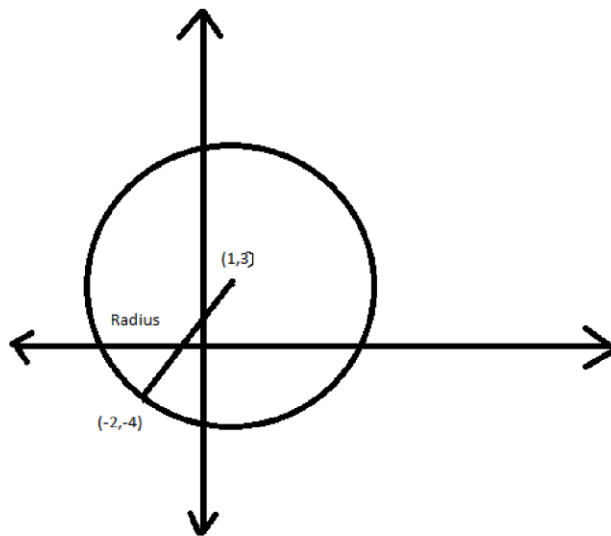
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$$d = \sqrt{(2-5)^2 + (4-6)^2}$$
$$= \sqrt{(-3)^2 + (-2)^2}$$
$$= \sqrt{9+4}$$
$$= \sqrt{13}$$

**Solution 2:**

It is given that center of the circle is $(1,3)$. We are also given a point on the circle that is $(-2,-4)$ as shown below.



The radius of the circle will be the distance between the points (1,3) and (-2,-4). That is

$$\text{Radius} = d = \sqrt{[1-(-2)]^2 + [3-(-4)^2}$$
$$= \sqrt{(3)^2 + (7)^2}$$
$$= \sqrt{9+49} = \sqrt{58}$$

**Solution 3:**

The general form of the equation of circle is given as

$$4x^2 + 4y^2 - 16x - 24y + 51 = 0$$

It can be re-written as

$$(4x^2 - 16x) + (4y^2 - 24y) = -51$$
$$(2x)^2 - 2(8x) + (2y)^2 - 2(12y) = -51$$

In order to complete the squares on the left hand side, we have to add 16 and 36 on both sides, it will then become

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$$(2x)^2 - 2(8x) + 16 + (2y)^2 - 2(12y) + 36 = -51 + 16 + 36$$
$$(2x)^2 - 2(2x)(4) + (4^2) + (2y)^2 - 2(2y)(6) + (6)^2 = 1$$
$$(2x - 4)^2 + (2y - 6)^2 = 1$$
$$(x - 2)^2 + (y - 3)^2 = \left(\frac{1}{4}\right)$$

Comparing it with the standard form of the equation, the center of the circle will be $(2, 3)$ and the radius will be $\frac{1}{2}$.

### Solution 4:
The general form of the equation of circle is given as
$$2x^2 + 2y^2 + 6x - 8y + 12 = 0$$
It can be re-written as
$$(2x^2 + 6x) + (2y^2 - 8y) = -12$$
$$(x^2 + 3x) + (y^2 - 4y) = -6$$

In order to complete the squares on the left hand side, we have to add $\frac{9}{4}$ and 4 on both sides, it will then become

$$(x^2 + 3x + \frac{9}{4}) + (y^2 - 4y + 4) = -6 + \frac{9}{4} + 4$$
$$(x^2 + 2(x)\left(\frac{3}{2}\right) + \left(\frac{3}{2}\right)^2 + (y)^2 - 2(y)(2) + (2)^2 = \frac{1}{4}$$
$$\left(x + \frac{3}{2}\right)^2 + \left(y - 2\right)^2 = \frac{1}{4}$$

Comparing it with the standard form of the equation, the center of the circle will be $\left(-\frac{3}{2}, 2\right)$ and radius will be $\frac{1}{2}$.

### Solution 5:

The general form of the equation of circle is given as
$$x^2 + y^2 - 4x - 6y + 8 = 0$$
This can be re-written as
$$(x^2 - 4x) + (y^2 - 6y) = -8$$
In order to complete the squares on the left hand side, we have to add 4 and 9 on both sides, it will then become

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$$(x^2 - 4x + 4) + (y^2 - 6y + 9) = -8 + 4 + 9$$
$$(x)^2 - 2(x)(2) + (2)^2 + (y)^2 - 2(y)(3) + (3)^2 = 5$$
$$(x-2)^2 + (y-3)^2 = 5$$

Comparing it with the standard form of the equation, the center of the circle will be (2,3) and the radius will be $\sqrt{5}$.

**SOLUTION 6:**

$$\because 3x^2 + 6x + 3y^2 + 18y - 6 = 0,$$
$$\Rightarrow 3(x^2 + 2x + y^2 + 6y - 2) = 0, \quad (\because \text{ taking 3 as common})$$
$$\Rightarrow x^2 + 2x + y^2 + 6y - 2 = 0, \quad (\because \text{ dividing by 3 on both sides})$$
$$\Rightarrow x^2 + 2x + 1 + y^2 + 6y + 9 = 2 + 9 + 1,$$
$$\Rightarrow (x+1)^2 + (y+3)^2 = 12,$$
$$\Rightarrow (x+1)^2 + (y+3)^2 = (\sqrt{12})^2,$$
$$\Rightarrow (x-(-1))^2 + (y-(-3))^2 = (\sqrt{12})^2,$$
$$\therefore \text{ Centre of the circle is (-1,-3) and radius is } \sqrt{12}.$$

**SOLUTION 7:**

$$x^2 - 6x + y^2 - 8y = 0, \quad (\because \text{ rearranging the term})$$
$$x^2 - 6x + y^2 - 8y + (3)^2 = (3)^2, \quad (\because \text{ adding } (3)^2 \text{ on both sides})$$
$$(x^2 - 6x + 9) + y^2 - 8y = 9,$$
$$(x^2 - 6x + 9) + y^2 - 8y + (4)^2 = 9 + (4)^2, \quad (\because \text{ adding } (4)^2 \text{ on both sides})$$
$$(x^2 - 6x + 9) + (y^2 - 8y + 16) = 9 + 16,$$
$$(x-3)^2 + (y-4)^2 = 9 + 16,$$
$$(x-3)^2 + (y-4)^2 = (\sqrt{25})^2, \quad \_\_\_\_\_ \text{ eq.}(1)$$
$$\because (x-x_0)^2 + (y-y_0)^2 = r^2. \quad \_\_\_\_\_ \text{ eq.}(2)$$

The eq.(1) is now in the standard form of eq.(2). This equation represents a circle with the center at (3, 4) and with a radius equal to $\sqrt{25}$.

**SOLUTION 8:**

The standard form of equation of circle is

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$$(x-h)^2 + (y-k)^2 = r^2 \; ,$$

$Here \; h = 3 \; , \; k = -2 \; , \; r = 4 \; ,$

$$(x-3)^2 + (y-(-2))^2 = 4^2 \; ,$$

$$x^2 - 6x + 9 + y^2 + 4 + 4y = 16 \; ,$$

$$x^2 + y^2 - 6x + 4y = 16 - 9 - 4 \; ,$$

$$x^2 + y^2 - 6x + 4y = 3.$$

## SOLUTION 9:

The distance formula between two points $(x_1, y_1)$ and $(x_2, y_2)$ in a coordinate plane is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \; ,$$

$$d = \sqrt{(8-2)^2 + (6-4)^2} \; ,$$

$$= \sqrt{(6)^2 + (2)^2} \; ,$$

$$= \sqrt{36 + 4} \; ,$$

$$= \sqrt{40} \; ,$$

$$= 2\sqrt{10}.$$

## SOLUTION 10:

The radius is the distance between the center and any point on the circle, so find the distance:

$$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \; ,$$

$$r = \sqrt{(3-(-1))^2 + (-2-(-3))^2} \; ,$$

$$= \sqrt{(3+1)^2 + (-2+3)^2} \; ,$$

$$= \sqrt{(4)^2 + (1)^2} \; ,$$

$$= \sqrt{16 + 1} \; ,$$

$$= \sqrt{17} \; ,$$

$$\approx 4.123.$$

Then the radius is $\sqrt{17}$ , or about 4.123, rounded to three decimal places.

# Practice Exercise
# Lecture No.6: Functions

## Q.No.1

Find the natural domain and the range of the given function

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$h(x) = \cos^2(\sqrt{x})$.

**Solution:**

As we know that the $\sqrt{x}$ is defined on non-negative real numbers $x \geq 0$. This means that the natural domain of $h(x)$ is the set of positive real numbers.

Therefore, the natural domain of $h(x) = [0, +\infty)$.

As we also know that the range of trigonometric function $\cos x$ is $[-1, 1]$.

The function $\cos^2 \sqrt{x}$ always gives positive real values within the range 0 and 1 both inclusive. From this we conclude that the range of $h(x) = [0, 1]$.

## Q.No.2

Find the domain and range of function $f$ defined by $f(x) = x^2 - 2$.

**Solution:**

$\because \quad f(x) = x^2 - 2$

The domain of this function is the set of all real numbers.

The range is the set of values that $f(x)$ takes as $x$ varies. If $x$ is a real number, $x^2$ is either positive or zero. Hence we can write the following:

$x^2 \geq 0$,

Subtract $-2$ on both sides to obtain

$x^2 - 2 \geq -2$.

The last inequality indicates that $x^2 - 2$ takes all values greater than or equal to $-2$. The range of function $f$ is the set of all values of $f(x)$ in the interval $[-2, +\infty)$.

## Q.No.3

Determine whether $y = \pm\sqrt{x+3}$ is a function or not? Justify your answer.

**Solution:**

$\because \quad y = \pm\sqrt{x+3}$

This is not a function because each value that is assigned to '*x*' gives two values of *y*

So this is not a function. For example, if *x*=1 then

$y = \pm\sqrt{1+3}$ ,

$y = \pm\sqrt{4}$ ,

$y = \pm 2$.

## Q.No.4

Determine whether $y = \dfrac{x+2}{x+3}$ is a function or not? Justify your answer.

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**Solution:**

$$\because \quad y = \frac{x+2}{x+3}$$

This is a function because each value that is assigned to '$x$' gives only one value of $y$

So this is a function. For example if $x$=1 then

$$y = \frac{1+2}{1+3},$$

$$y = \frac{3}{4},$$

$$y = 0.75.$$

## Q.No.5

**(a)** Find the natural domain of the function $f(x) = \frac{x^2 - 16}{x - 4}$.

**(b)** Find the domain of function $f$ defined by $f(x) = \frac{-1}{(x+5)}$.

**Solution:**

**(a)**

$$\because \quad f(x) = \frac{x^2 - 16}{x - 4},$$

$$\Rightarrow \quad f(x) = \frac{(x+4)(x-4)}{(x-4)},$$

$$= (x+4) \quad ; \quad x \neq 4.$$

This function consists of all real numbers $x$, except $x = 4$.

**(b)**

$$\because \quad f(x) = \frac{-1}{(x+5)}$$

This function consists of all real numbers $x$, except $x = -5$. Since $x = -5$ would make the denominator equal to zero and the division by zero is not allowed in mathematics. Hence the domain in interval notation is given by $(-\infty, -5) \cup (-5, +\infty)$.

# Practice Exercise
# Lecture No.8: Graphs of Functions
# Lecture No.9: Limits

Choose the correct option for the following questions:

1)   If a vertical line insects the graph of the equation   at two points, then which of the following is true.

I.   It represents a function.

II.   It represents a parabola.

III. It represents a straight line.

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IV. It does not represent a function.

2) Which of the following is the reflection of the graph of   about y-axis?

I.

II.

III.

IV.

3) Given the graph of a function   and a constant c, the graph of   can be obtained by

I. Translating the graph of  up by c units.

II. Translating the graph of down by c units.

III. Translating the graph of  right by c units.

IV. Translating the graph of  left by c units.

4) Given the graph of a function   and a constant c, the graph of   can be obtained by

I. Translating the graph of   up by c units.

II. Translating the graph of  down by c units.

III. Translating the graph of   right by c units.

IV. Translating the graph of   left by c units.

5) Which of the following is the reflection of the graph of   about x-axis?

V.

VI.

VII.

VIII.

Q.No.6

If   and   then find the value of 'c' so that  .

Answer:

Q.No.7

Find the limit by using the definition of absolute value

Answer:

Q.No.8

Find the limit by using the definition of absolute value

Answer: -1

Q.No.9

Evaluate:  .

Answer: 0

Q.No.10

Evaluate:  .

Answer:

# Practice Exercise
## Lecture No.14: Rate of Change

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## Q.No.1
Find the instantaneous rate of change of $f(x) = x^2 + 1$ at $x_0$.

**Answer:** $2x_0$

## Solution:
Since $f(x) = x^2 + 1$ at $x_0$,

$\lim_{h\to 0} \frac{f(x_0+h)-f(x_0)}{h} = \lim_{h\to 0} \frac{((x_0+h)^2+1)-(x_0^2+1)}{h}$,

$= \lim_{h\to 0} \frac{x_0^2 + h^2 + 2x_0 h + 1 - x_0^2 - 1}{h}$,

$= \lim_{h\to 0} \frac{h^2 + 2x_0 h}{h}$,

$= \lim_{h\to 0} \frac{h(h+2x_0)}{h}$,

$= \lim_{h\to 0} (h + 2x_0)$,

$= 2x_0$ by applying limit, (Answer).

## Q.No.2
Find the instantaneous rate of change of $f(x) = \sqrt{x+2}$ at an arbitrary point of the domain of $f$.

**Answer:** $\frac{1}{2\sqrt{a+2}}$

## Solution:
Let $a$ be any arbitrary point of the domain of $f$. The instantaneous rate of change of $f(x)$ at $x = a$ is

$\lim_{x\to a} \frac{f(x)-f(a)}{x-a} = \lim_{x\to a} \frac{\sqrt{x+2}-\sqrt{a+2}}{x-a}$,

$= \lim_{x\to a} \frac{\sqrt{x+2}-\sqrt{a+2}}{x-a} \times \frac{\sqrt{x+2}+\sqrt{a+2}}{\sqrt{x+2}+\sqrt{a+2}}$ by rationalizing,

$= \lim_{x\to a} \frac{x+2-a-2}{(x-a)\sqrt{x+2}+\sqrt{a+2}}$,

$= \lim_{x\to a} \frac{x-a}{(x-a)\sqrt{x+2}+\sqrt{a+2}}$,

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$$= \lim_{x \to a} \frac{1}{\sqrt{x+2} + \sqrt{a+2}},$$

$$= \frac{1}{\sqrt{a+2} + \sqrt{a+2}} \text{ by applying limit,}$$

$$= \frac{1}{2\sqrt{a+2}} \text{ (Answer).}$$

## Q.No.3

The distance traveled by an object at time $t$ is $= f(t) = t^2$. Find the instantaneous velocity of the object at $t_0 = 4\ sec$.

**Answer:** 8

## Solution:

$$v_{inst} = m_{tan} = \lim_{t_1 \to t_0} \frac{f(t_1) - f(t_0)}{t_1 - t_0},$$

$$= \lim_{t_1 \to t_0} \frac{t_1^2 - 4^2}{t_1 - t_0},$$

$$= \lim_{t_1 \to t_0} \frac{t_1^2 - 16}{t_1 - t_0},$$

$$= \lim_{t_1 \to t_0} \frac{(t_1 + 4)(t_1 - 4)}{t_1 - t_0},$$

$$= \lim_{t_1 \to 4} \frac{(t_1 + 4)(t_1 - 4)}{(t_1 - 4)} \text{ because } t_0 = 4\ sec,$$

$$= \lim_{t_1 \to 4} (t_1 + 4),$$

$$= 4 + 4 \text{ by applying limit,}$$

$$= 8 \text{ (Answer).}$$

## Q.No.4

Find the instantaneous rate of change of $f(x) = x^3 + 1$ at $x_0 = 2$.

**Answer:** 12

## Solution:

$$\lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h} = \lim_{h \to 0} \frac{f(2 + h) - f(2)}{h},$$

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$$= \lim_{h \to 0} \frac{((2+h)^3+1)-(2^3+1)}{h},$$

$$= \lim_{h \to 0} \frac{(2^3+3(2)^2h+3(2)h^2+h^3+1)-(2^3+1)}{h},$$

$$= \lim_{h \to 0} \frac{8+12h+6h^2+h^3+1-(8+1)}{h},$$

$$= \lim_{h \to 0} \frac{9+12h+6h^2+h^3-9}{h},$$

$$= \lim_{h \to 0} \frac{12h+6h^2+h^3}{h},$$

$$= \lim_{h \to 0} \frac{h(12+6h+h^2)}{h},$$

$$= \lim_{h \to 0} (12+6h+h^2),$$

$$= 12 \quad \text{(Answer)}.$$

## Q.No.5

(a) The distance traveled by an object at time $t$ is $s = f(t) = t^2$. Find the average velocity of the object between $t = 2\ sec.$ and $t = 4\ sec.$

(b) Let $f(x) = \frac{1}{x-1}$. Find the average rate of change of $f$ over the interval [5,7].

**Answer: (a)** $14$ **(b)** $\frac{5}{2}\ m/sec$

## Solution:

(a) Avergae Velocity $= \frac{Distance\ travelled\ during\ interval}{TIme\ Elapsed}$,

$$v_{ave} = \frac{f(t_1)-f(t_0)}{t_1-t_0},$$

$$= \frac{f(4)-f(2)}{4-2},$$

$$= \frac{4^2-2^2}{2},$$

$$= \frac{16-4}{2},$$

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$$= \frac{12}{2},$$

$$= 6 \quad (\text{Answer}).$$

**(b)** Avergae Velocity $= \dfrac{Distance\ travelled\ during\ interval}{TIme\ Elapsed}$,

$$m_{sec} = \frac{f(x_1) - f(x_0)}{x_1 - x_0},$$

$$= \frac{f(7) - f(5)}{7 - 5},$$

$$= \frac{\frac{1}{7-1} - \frac{1}{5-1}}{2},$$

$$= \frac{\frac{1}{6} - \frac{1}{4}}{2},$$

$$= -\frac{1}{24} \ m/sec. \quad (\text{Answer}).$$

## Practice Exercise
## Lecture No.17: Derivatives of Trigonometric Functions
## Q.No.1

Find $\frac{dy}{dx}$ if $y = x^3 \cot x - \frac{3}{x^3}$.

**Answer:** $3x^2 \cot x - x^3 cosec^2 x + \frac{9}{x^4}$

**Solution:**

Given $y = x^3 \cot x - \frac{3}{x^3}$,

$\frac{dy}{dx} = \cot x \frac{d}{dx}(x^3) + x^3 \frac{d}{dx}(\cot x) - \frac{d}{dx}\left(\frac{3}{x^3}\right)$,

$= \cot x\,(3x^2) + x^3(-cosec^2 x) - 3\frac{d}{dx}\left(\frac{1}{x^3}\right)$,

$= 3x^2 \cot x - x^3 cosec^2 x + \frac{9}{x^4} \quad (\text{Answer}).$

## Q.No.2

Find $\frac{dy}{dx}$ if $y = x^4 \sin x$ at $x = \pi$.

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## Answer: $-\pi^4$
## Solution:

$\because \dfrac{d}{dx}(f \cdot g) = f \cdot \dfrac{d}{dx}(g) + g \cdot \dfrac{d}{dx}(f)$,

$y = x^4 \sin x$ at $x = \pi$,

$\dfrac{d}{dx} = \sin x \dfrac{d}{dx}(x^4) + x^4 \dfrac{d}{dx}(\sin x)$,

$\quad = \sin x\,(4x^3) + x^4(\cos x)$,

$\quad = 4x^3 \sin x + x^4 \cos x$,

$\quad = 4\pi^3 \sin \pi + \pi^4 \cos \pi,\ \text{at } x = \pi$,

$\quad = 4\pi^3(0) + \pi^4(-1)$,

$\quad = -\pi^4 \quad$ (Answer).

## Q.No.3

Find $f'(t)$ if $f(t) = \dfrac{2-8t+t^2}{\sin t}$.

## Answer: $\dfrac{[(2t-8)(\sin t)] - [(t^2 - 8t + 2)(\cos t)]}{\sin^2 t}$

## Solution:

$\because \dfrac{d}{dx}\left(\dfrac{f(x)}{g(x)}\right) = \dfrac{g(x) \cdot \frac{d}{dx}(f(x)) - f(x) \cdot \frac{d}{dx}(g(x))}{[g(x)]^2}$,

$f(t) = \dfrac{2-8t+t^2}{\sin t}$,

$f'(t) = \dfrac{[(\sin t)(-8+2t)] - [(2-8t+t^2)(\cos t)]}{(\sin t)^2}$,

$\quad = \dfrac{[(2t-8)(\sin t)] - [(t^2 - 8t + 2)(\cos t)]}{\sin^2 t} \quad$ (Answer).

## Q.No.4

Find $f'(y)$ if $(y) = \dfrac{\sin y + 3\tan y}{y^3 - 2}$.

## Answer: $\dfrac{[(y^3-2)(\cos y + 3\sec^2 y)] - [(\sin y + 3\tan y) + (3y^2)]}{y^6 - 4y^3 + 4}$

## Solution:

$\dfrac{d}{dx}\left(\dfrac{f(x)}{g(x)}\right) = \dfrac{g(x) \cdot \frac{d}{dx}(f(x)) - f(x) \cdot \frac{d}{dx}(g(x))}{[g(x)]^2}$,

$f(y) = \dfrac{\sin y + 3\tan y}{y^3 - 2}$.

$f'(y) = \dfrac{[(y^3-2)(\cos y + 3\sec^2 y)] - [(\sin y + 3\tan y) + (3y^2)]}{(y^3-2)^2}$,

$\quad = \dfrac{[(y^3-2)(\cos y + 3\sec^2 y)] - [(\sin y + 3\tan y) + (3y^2)]}{y^6 - 4y^3 + 4} \quad$ (Answer).

## Q.No.5

(a) Find $\dfrac{dy}{dx}$ if $y = (5x^2 + 3x + 3)(\sin x)$.

(b) Find $f'(t)$ if $(t) = 5t \sin t$.

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**Answer:** (a) $(5 x^2 + 3x + 3)(\cos x) + \sin x \cdot (10x + 3)$

(b) $5 t \cos t + 5 \sin t$

**Solution:**

(a) $\because \frac{d}{dx}(f \cdot g) = f \cdot \frac{d}{dx}(g) + g \cdot \frac{d}{dx}(f),$

$y = (5x^2 + 3x + 3)(\sin x)$,

$\frac{d}{dx}[(5 x^2 + 3x + 3)(\sin x)] = (5 x^2 + 3x + 3)(\cos x) + \sin x (10x + 3)$    (Answer).

(b) $\because \frac{d}{dx}(f \cdot g) = f \cdot \frac{d}{dx}(g) + g \cdot \frac{d}{dx}(f),$

$f(t) = 5t \sin t$,

$\frac{d}{dx}(5 t \sin t) = 5 t \cos t + (\sin t)(5),$

$= 5 t \cos t + 5 \sin t$    (Answer).

# Practice Exercise
## Lecture No.20: Derivatives of Logarithmic and Exponential Functions

## Q.No.1

Differentiate: $y = (5 - x)^{\sqrt{x}}$.

**Answer:** $\frac{dy}{dx} = \left( \frac{\ln(5-x)}{2\sqrt{x}} - \frac{\sqrt{x}}{(5-x)} \right) \cdot (5-x)^{\sqrt{x}}$

**Solution:**

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$$\because \quad y = (5-x)^{\sqrt{x}} ,$$

taking log on both sides ,

$$\Rightarrow \ln y = \sqrt{x} \ \ln(5-x) \qquad \left( \because \ln m^n = n \ln m \right) ,$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1}{2\sqrt{x}}.\ln(5-x) + \frac{1}{(5-x)}(-1).\sqrt{x} \ ,$$

$$\Rightarrow \frac{dy}{dx} = \left( \frac{\ln(5-x)}{2\sqrt{x}} - \frac{\sqrt{x}}{(5-x)} \right).y \ ,$$

$$\Rightarrow \frac{dy}{dx} = \left( \frac{\ln(5-x)}{2\sqrt{x}} - \frac{\sqrt{x}}{(5-x)} \right).(5-x)^{\sqrt{x}} .$$

# Q.No.2

Differentiate $y = (\cos x)^{8x}$ with respect to '$x$'.

**Answer:** $\dfrac{dy}{dx} = \left( 8 \ln(\cos x) - \dfrac{8x\sin x}{\cos x} \right)(\cos x)^{8x}$

# Solution:

$$\because \quad y = (\cos x)^{8x} ,$$

taking log on both sides,

$$\Rightarrow \ln y = (8x)\ln(\cos x) , \qquad\qquad \left( \because \ln m^n = n \ln m \right),$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = 8.\ln(\cos x) + \frac{1}{(\cos x)}.(-\sin x).(8x), \qquad \left( \because \frac{d}{dx}(\ln x) = \frac{1}{x} \ ; \ \frac{d}{dx}(\cos x) = -\sin x \right),$$

$$\Rightarrow \frac{dy}{dx} = \left( 8 \ln(\cos x) - \frac{8x\sin x}{\cos x} \right).y,$$

$$\Rightarrow \frac{dy}{dx} = \left( 8 \ln(\cos x) - \frac{8x\sin x}{\cos x} \right)(\cos x)^{8x} .$$

# Q.No.3

Differentiate $y = x^{\sin 5x}$ with respect to '$x$'.

**Answer:** $\dfrac{dy}{dx} = \left( 5(\cos 5x).\ln(x) + \dfrac{\sin 5x}{x} \right)\left( x^{\sin 5x} \right)$

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## Solution:

$\because \quad y = x^{\sin 5x}$ ,

Taking log on both sides ,

$\Rightarrow \ln y = (\sin 5x)\ln(x)$ , $\qquad (\because \ln m^n = n \ln m)$ ,

$\Rightarrow \dfrac{1}{y}\dfrac{dy}{dx} = 5(\cos 5x).\ln(x) + \dfrac{1}{x}.(\sin 5x)$ , $\qquad \left(\because \dfrac{d}{dx}(\ln x) = \dfrac{1}{x} \; ; \; \dfrac{d}{dx}(\sin x) = \cos x\right)$ ,

$\Rightarrow \dfrac{dy}{dx} = \left(5(\cos 5x).\ln(x) + \dfrac{\sin 5x}{x}\right).y$ ,

$\Rightarrow \dfrac{dy}{dx} = \left(5(\cos 5x).\ln(x) + \dfrac{\sin 5x}{x}\right)\left(x^{\sin 5x}\right)$ .

## Q.No.4

Differentiate $y = x\,e^{3x+4}$ .

**Answer:** $\dfrac{dy}{dx} = e^{3x+4} + 3xe^{3x+4}$

## Solution:

$\because \quad y = x\,e^{3x+4}$ ,

$\Rightarrow \dfrac{dy}{dx} = e^{3x+4} + xe^{3x+4}\dfrac{d}{dx}(3x+4)$ ,

$\Rightarrow \dfrac{dy}{dx} = e^{3x+4} + 3xe^{3x+4}$ .

## Q.No.5

Find the derivative of the function $y = \ln(2 + x^5)$ with respect to '$x$'.

**Answer:** $\dfrac{dy}{dx} = \dfrac{5x^4}{(2 + x^5)}$

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## Solution:

$y = \ln(2 + x^5)$ ,

now taking the derivative of the function on both sides ,

$$\frac{dy}{dx} = \frac{d}{dx}\{\ln(2 + x^5)\} \ ,$$

$$\frac{dy}{dx} = \frac{1}{(2 + x^5)}\frac{d}{dx}(2 + x^5) \ ,$$

$$\frac{dy}{dx} = \frac{1}{(2 + x^5)}(0 + 5x^4) \ ,$$

$$\frac{dy}{dx} = \frac{5x^4}{(2 + x^5)} \ .$$

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